Matchings with Externalities and Attitudes

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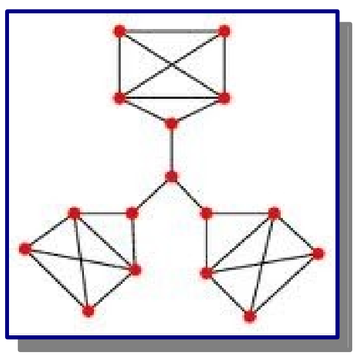
Matchings

Intensely studied class of combinatorial problems:

One-to-One: The stable marriage problem

One-to-Many: House allocation problems, assigning medical interns to hospitals

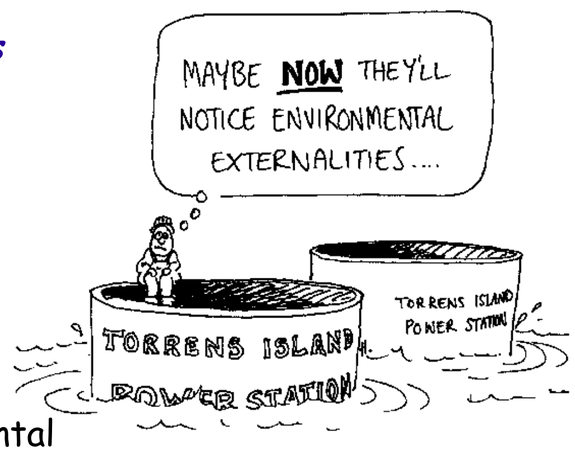
Many-to-Many: Most labor markets, friendships



Externalities

Also known as ***transaction spillovers***

Third parties are influenced by transactions they did not agree to



Positive externalities: Education, immunization, environmental remediation, research

Negative externalities: Environmental pollution, smoking, alcohol consumption and car accidents

Externalities in Matchings

Matchings are a natural model for studying externalities:

- Agents are influenced not only by their own choices (matches), but also by the transactions that others make

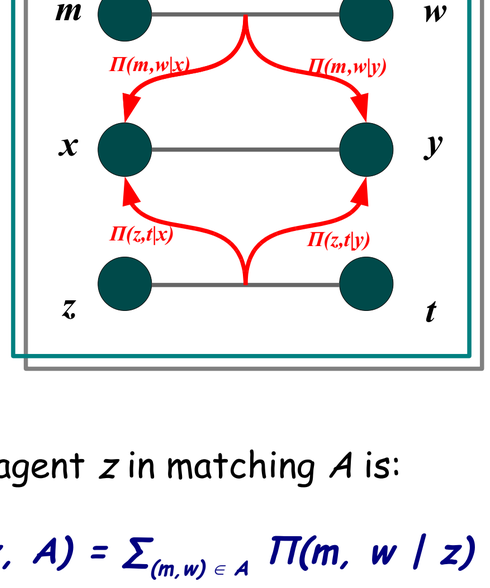
- In general, agents can have a different utility for every different state of the world

This work: Succinct model of externalities in matchings (polynomial-size preferences in the number of agents)

Model

Matching game: $G = (M, W, \Pi)$, where M and W are agents on the two sides of the market

- Denote by $\Pi(m, w \mid z)$ the influence of match (m, w) on agent z (if the match forms)



- The utility of an agent z in matching A is:

$$u(z, A) = \sum_{(m,w) \in A} \Pi(m, w \mid z)$$

Stability

Stability is a central question in game theoretic analyses of matchings

- Which matchings are such that the agents don't have incentives to ***(i) cut existing matches*** or ***(ii) form new matches***?

The stable outcomes depend on the solution concept used

- ***This work:*** pairwise stability and the core

Deviation: Each member of a deviating coalition B must either sever a match with a player in N, or form a new match with a player in B

Response to a deviation: Given matching A and deviation A' of coalition B, the response $\Gamma(B, A, A')$ defines the reaction of the players outside B upon the deviation

Stability: A matching is stable if no coalition can deviate and improve the utility of at least one member while not degrading the other members in the response of $N \setminus B$

How will society react to a deviation?

- The agents need to compute the response (possibly intractable)

Attitudes (Heuristics)

Optimism: Deviators assume the best case reaction from the rest of the agents (attitude à la *"All is for the best in the best of all the possible worlds"*)

Neutrality: No reaction (the deviators assume the others are not going to do anything about it)

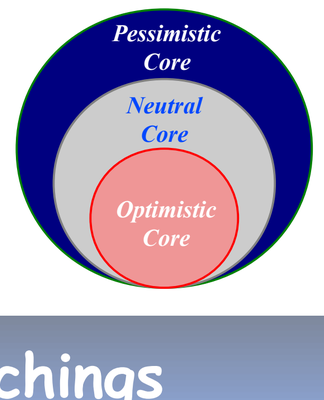
Pessimism: Worst case reaction (deviators assume the remaining agents will retaliate in the worst possible way

Many others possible: Contractual: Assume retaliation from players hurt by the deviation, and no reaction from the rest

Many-to-Many Matchings

Core	*Optimism*	*Neutrality*	*Pessimism*
Membership	*P*	*coNP-complete*	*coNP-complete*
Nonemptiness	*NP-complete*	*NP-hard*	*NP-hard*

The cores are included in each other:



One-to-One Matchings

Pairwise Stable Set	*Optimism*	*Neutrality*	*Pessimism*
Membership	*P*	*P*	*P*
Nonemptiness	*NP-complete*	*P*	*P*

Core	*Optimism*	*Neutrality*	*Pessimism*
Membership	*P*	*coNP-complete*	*coNP-complete*
Nonemptiness	*NP-complete*	*NP-hard*	*NP-hard*