Matchings with Externalities and Attitudes

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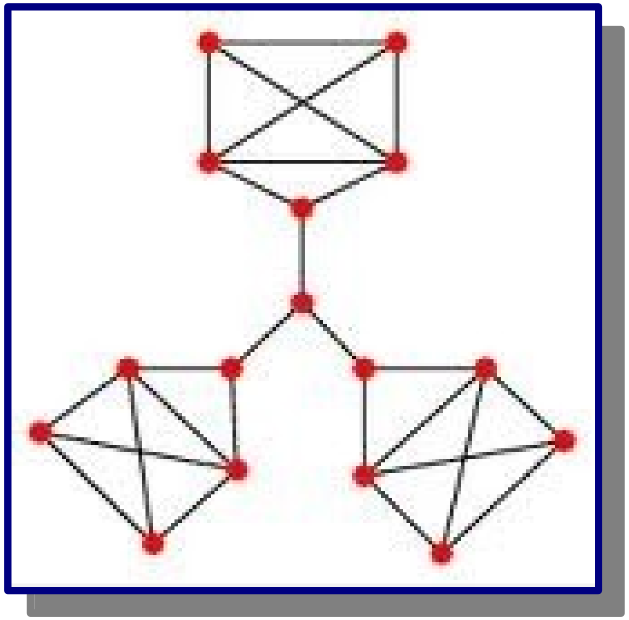
Matchings

Intensely studied class of combinatorial problems:

One-to-One: The stable marriage problem

One-to-Many: House allocation problems, assigning medical interns to hospitals



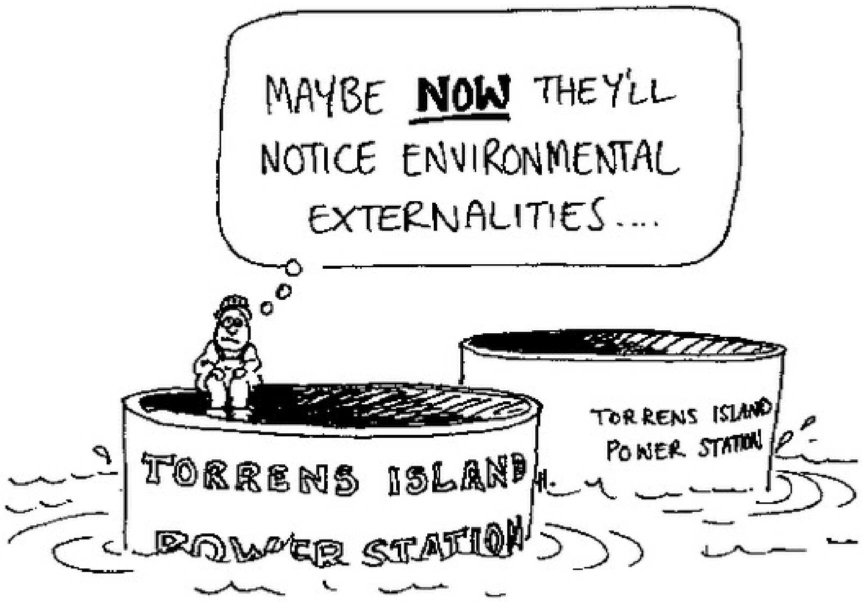
Many-to-Many: Most labor markets, friendships

Externalities

Also known as ***transaction spillovers***

Third parties are influenced
by transactions they did not agree to

Positive externalities: Education, immunization, environmental cleanup, research



Negative externalities: Environmental pollution, smoking, drinking and driving

Externalities in Matchings

Matchings are a natural model for studying externalities

Agents influenced not only by their own choices (matches), but also by the choices that other agents make

Existing work in economics assumes agents have a different utility for every state of the world

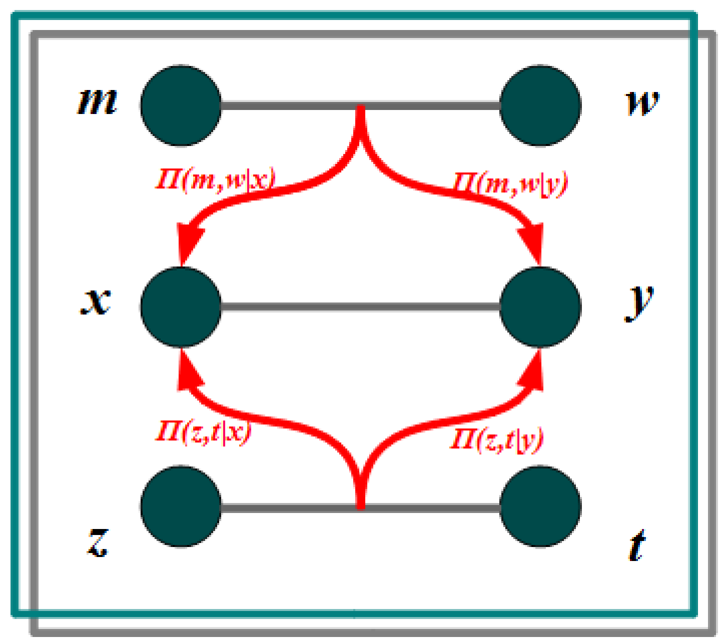
Can bounded rational agents reason about such games?

- Succinct model of externalities in matchings (polynomial-size preferences in the number of agents)

Model

Let G = (M, W, Π) be a matching game, where M and W are agents on the two sides of the market

Denote by Π(m, w | z) the influence of match (m, w) on agent z (if the match forms)



The utility of an agent z in matching A is:

$$u(z, A) = \sum_{(m,w)\in A} \Pi(m, w|z)$$

Model

Stability is a central question in game theoretic analyses of matchings

Given a game, which matchings are such that the agents don't have incentives to ***(i) cut existing matches*** or ***(ii) form new matches***?

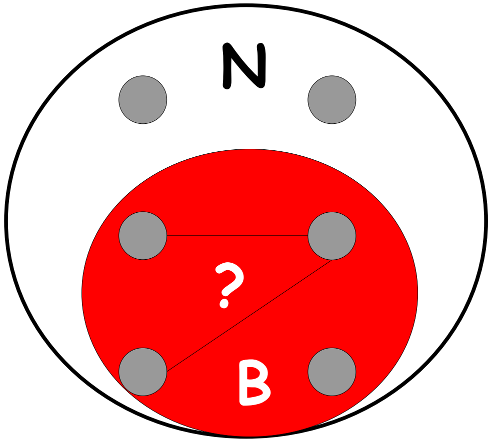
The stable outcomes depend on the solution concept used

- ***This work:*** pairwise stability and the core

Solution Concept

Core Stability

Given a matching game G = (M, W, Π), a matching A of G is core-stable if there does not exist a set of agents B ⊆ N, which can deviate and improve the utility of at least one member of B while not degrading the others.



Solution Concept

Deviation

Each member of a deviating coalition B must perform some action: either sever a match with an agent in N, or form a new match with an agent in B

Response

Given matching A and deviation A' of coalition B, the response $\Gamma(B, A, A')$ defines the reaction of the agents outside B upon the deviation

Solution Concept

Stability

A matching is stable if no coalition can deviate and improve the utility of at least one member while not degrading the other members in the response of $N \setminus B$

How will society respond to a deviation?

- The deviators need to estimate the response of the residual agents (which may be intractable)

Attitudes

Optimism: Deviators assume the best case reaction from the rest of the agents; hoping for the formation of matches good for the deviators and removal of all bad matches (attitude à la *"All is for the best in the best of all the possible worlds"*)

Neutrality: No reaction (the deviators behave as if the others are not going to do anything about the deviation)

Pessimism: Worst case reaction (deviators assume the remaining agents will retaliate in the worst possible way)

Attitudes

Many other definitions possible:

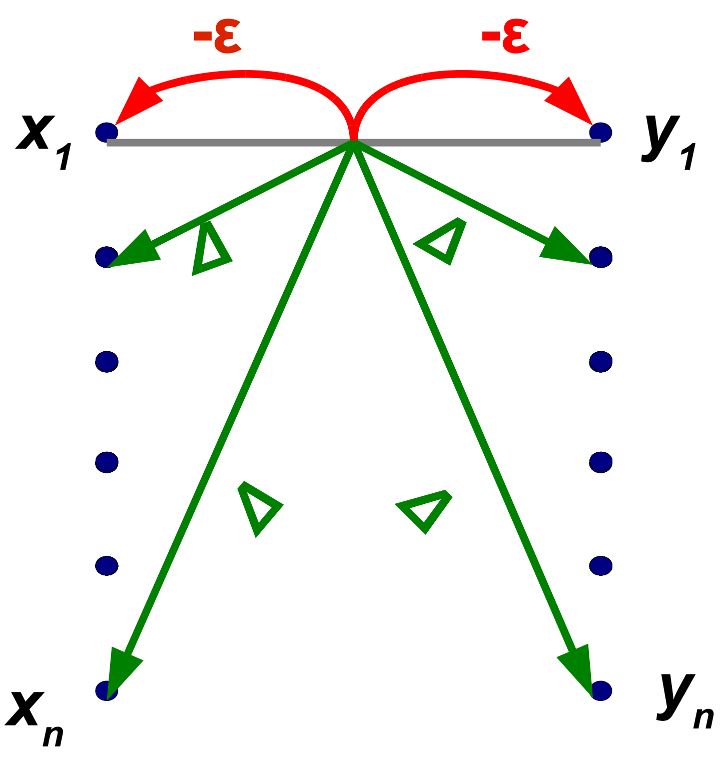
Contractual: Assume retaliation from agents hurt by the deviation, and no reaction from the rest

Recursive core (Koczy): when a coalition deviates, the residual agents react rationally (maximize their own payoff in the response)

Many-to-Many Matchings

Empty Neutral Core

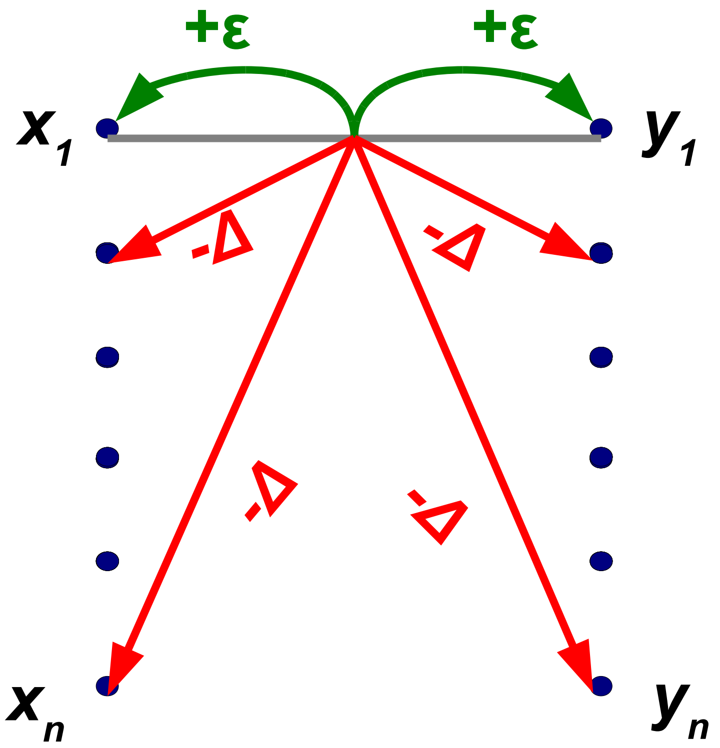
- The complete matching is Pareto optimal, but unstable
- The empty matching may be stable depending on *ε, Δ*



Many-to-Many Matchings

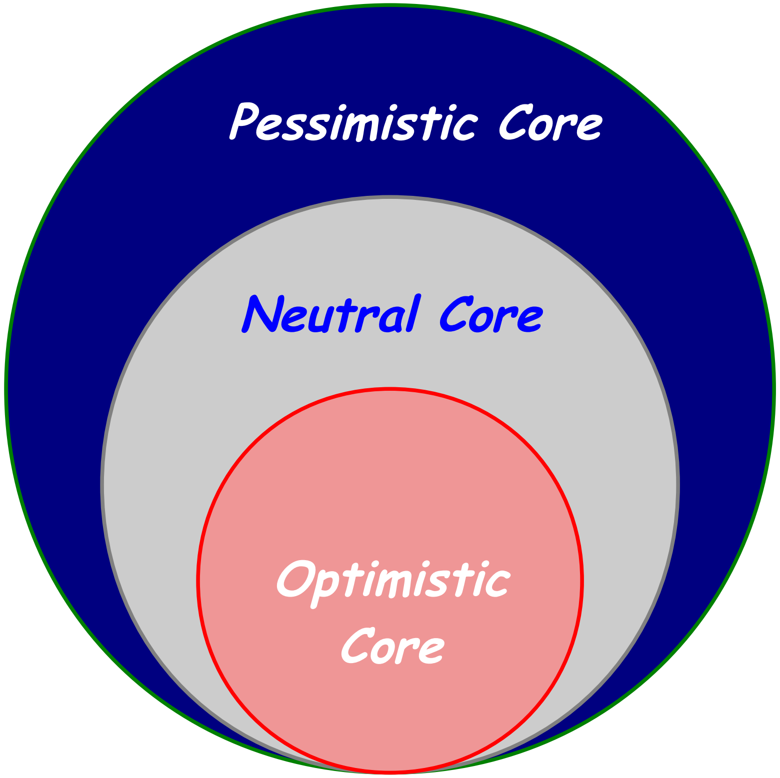
Empty Neutral Core (II)

- The complete matching is a tragic outcome for everyone; may be stable depending on *ε, Δ*



Many-to-Many Matchings

The cores are included in each other



Many-to-Many Matchings

Core	*Optimism*	*Neutrality*	*Pessimism*
Membership	*P*	*coNP-complete*	*coNP-complete*
Nonemptiness	*NP-complete*	*NP-hard*	*NP-hard*

Many-to-Many Matchings

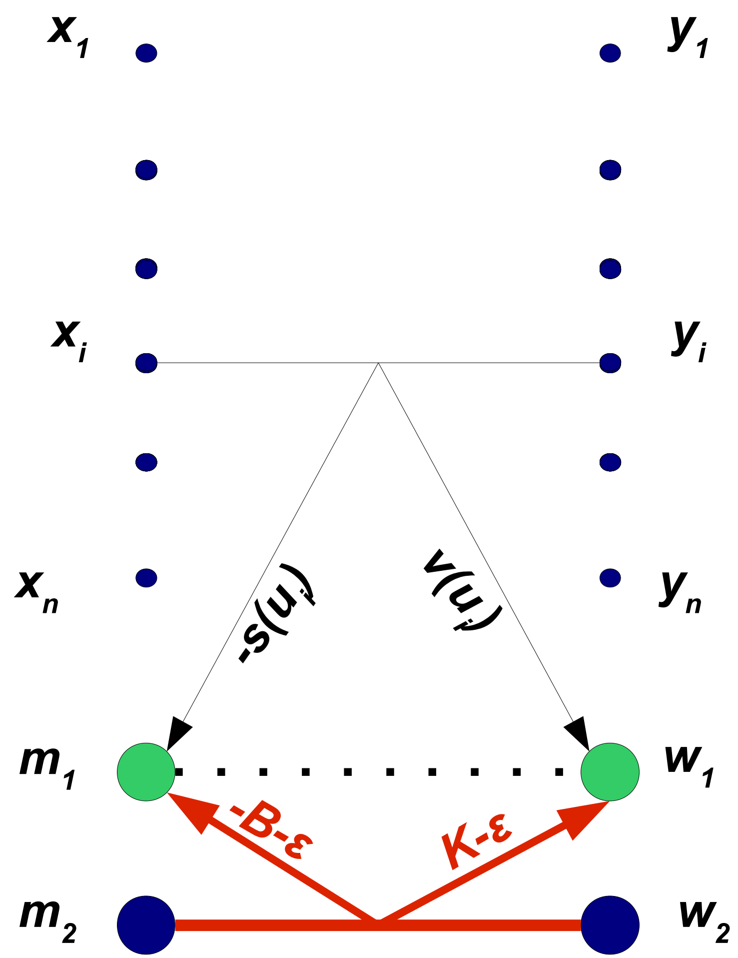
Theorem: Checking membership to the neutral core is *coNP*-complete.

Proof (sketch):

- Show the complementary problem is *NP*-complete
- Given $I = (U, s, v, B, K)$, construct game $G = (M, W, \Pi)$ and matching A such that A has a blocking coalition if and only if I has a solution

Many-to-Many Matchings

- $A = \{(m_2, w_2), (m_1), (w_1), (x_1), \ldots, (x_n), (y_1), \ldots, (y_n)\}$ has a blocking coalition ↔ I has a solution



One-to-One Matchings

Known as the stable marriage problem

- the Gale-Shapley algorithm used to compute stable outcomes

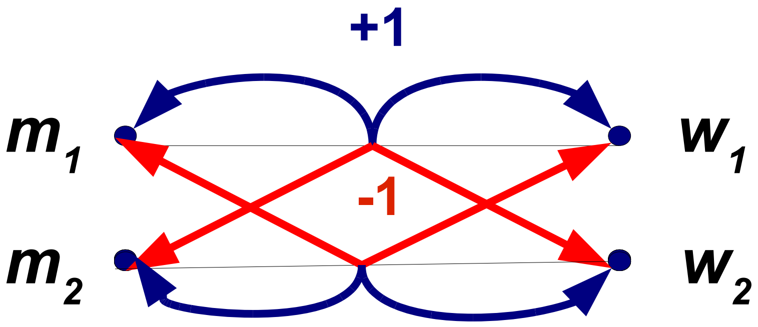
The Core with Externalities:

- Without externalities, the core is equivalent to the pairwise stable set
- The equivalence between pairwise stability and the core no longer holds with externalities

One-to-One Matchings with Externalities

- Moreover, under arbitrary Π values, even a pairwise stable solution does not always exist

Empty Neutral Pairwise Stable Set



One-to-One Matchings with Externalities

However, a pairwise stable matching under neutrality and pessimism always exists when Π is non-negative.

- Run Gale-Shapley by ignoring externalities and breaking ties arbitrarily

One-to-One Matchings with Externalities

Pairwise Stable Set	Optimism	Neutrality	Pessimism
Membership	P	P	P
Nonemptiness	NP-complete	P	P

Core	Optimism	Neutrality	Pessimism
Membership	P	coNP-complete	coNP-complete
Nonemptiness	NP-complete	NP-hard	NP-hard

Discussion

More refined solution concepts – interesting line of work in economics (e.g. the recursive core)

Externalities in social networks

- On platforms such as Facebook, agents are influenced by the matchings of others (friendships, subscriptions)

- Such cumulative effects can be expressed with additive models, but what is the right solution concept for bounded rational agents in such settings?